2 | Basic Operations

2.1 | Place Value

This section deals with the revision of place value. Remember that we write decimal numbers in the form:

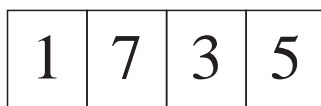| *Thousands*	*Hundreds*	*Tens*	*Units*	•	*Tenths*	*Hundredths*	*Thousandths*

Example 1

Here are some number cards:

$$\boxed{1} \quad \boxed{7} \quad \boxed{3} \quad \boxed{5}$$

You can use each card *once* to make the number 1735, like this:

$$\boxed{1\,|\,7\,|\,3\,|\,5}$$

(a) What is the *biggest* number you can make with the four cards?

(b) Explain why you *cannot* make an *even* number with the four cards.

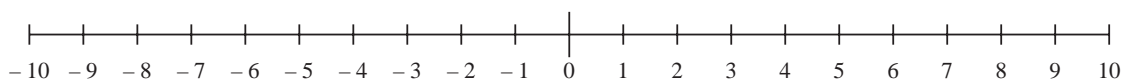(KS3/99/Ma/Tier 3-5/P2)

Solution

(a) The biggest number, using all four cards, is

$$7531$$

   (this is because $7 > 5 > 3 > 1$).

(b) To make an even number, the last digit must be even, but all four cards in this example show odd digits.

Note: It is often helpful to refer to a number line when comparing values; a number line can also show negative values:

```
 ├──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┤
-10 -9  -8  -7  -6  -5  -4  -3  -2  -1   0   1   2   3   4   5   6   7   8   9  10
```

Remember that the symbol $<$ means 'less than' and $>$ means 'greater than'.

Example 2

Put the correct sign, $<$ or $=$ or $>$, into each sentence.

(a) -7 .... -2

(b) $3 - 2$ .... -5

(c) $3 - 5$ .... $4 - 6$

Solution

(a)   From the number line shown, $-7 < -2$.

(b)   Since $3 - 2 = 1$, the comparison is 1 ... -5, so that $1 > -5$ (see number line).

(c)   Here we compare -2 ..... -2, giving $-2 = -2$.

Example 3

The arrow on this thermometer shows a temperature of $10 \,^{\circ}\text{C}$.

(a)   Draw an arrow on the thermometer to show a temperature of $24 \,^{\circ}\text{C}$.

   Label the arrow $24 \,^{\circ}\text{C}$.

(b)   Draw an arrow on the thermometer to show a temperature of $-4 \,^{\circ}\text{C}$.

   Label the arrow $-4 \,^{\circ}\text{C}$.

(c)   The temperature was $-10 \,^{\circ}\text{C}$.

   It went *up* $15 \,^{\circ}\text{C}$.

   What is the temperature now?

(d)   Write these temperatures in order, *coldest first.*

   $3 \,^{\circ}\text{C}$, $-10 \,^{\circ}\text{C}$, $0 \,^{\circ}\text{C}$, $20 \,^{\circ}\text{C}$, $-1 \,^{\circ}\text{C}$

2.1

Solution

(a)

24 ° C

10 ° C

− 4 ° C

(b)
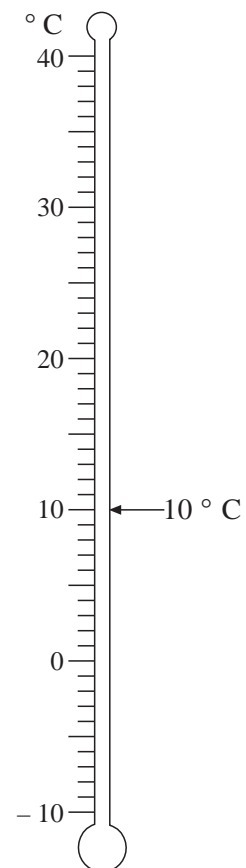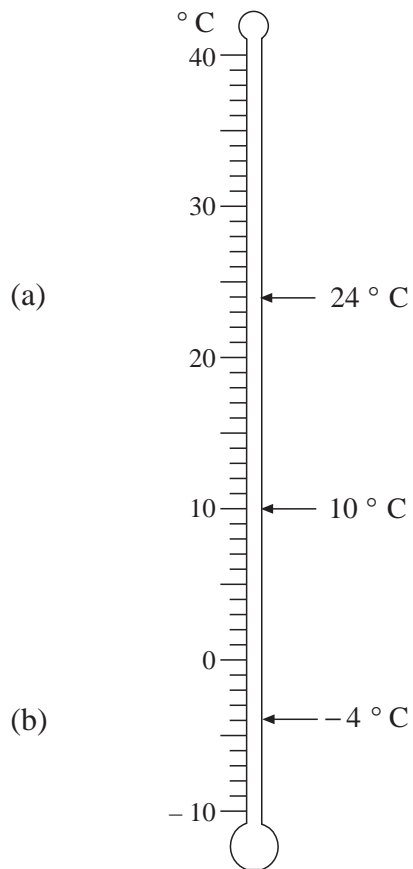
(c) $-10 \degree \text{C} + 15 \degree \text{C} = 5 \degree \text{C}$

(d) $-10 \degree \text{C}, \ -1 \degree \text{C}, \ 0 \degree \text{C}, \ 3 \degree \text{C}, \ 20 \degree \text{C}$

Exercises

1.   (a)   Write the numbers:

        (i)   one hundred and eighty,

        (ii)   two hundred and twelve,

        (iii) one hundred and eight,

        (iv)   ninety two

     (b)   Using the numbers in (a), write them in order with the smallest first.

2. Ali drew a picture to show what there is above and below the sea at Aber.
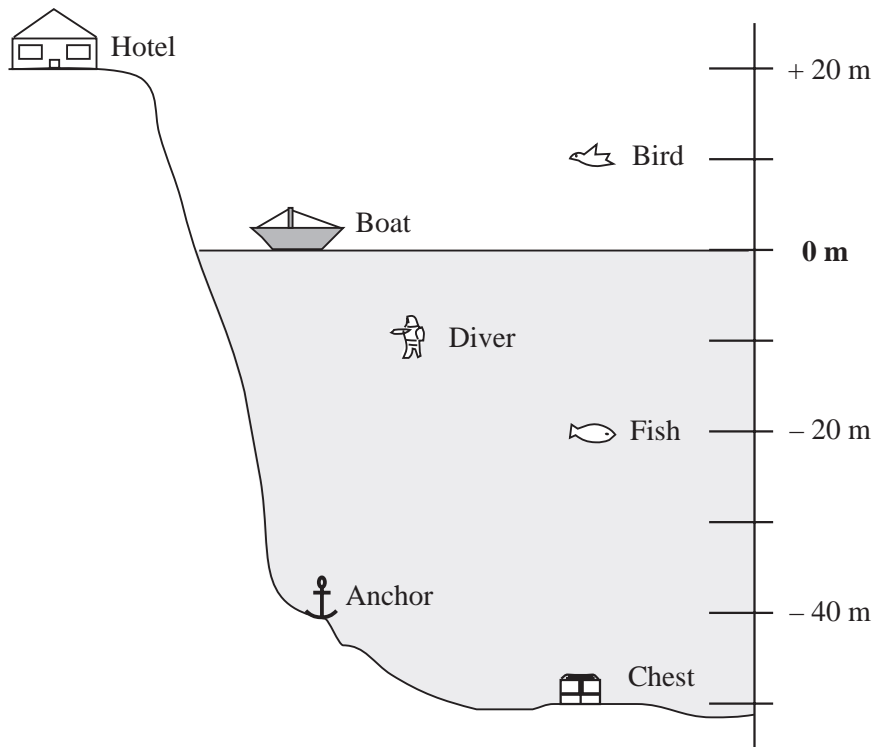
The anchor is at about -40 m.

(a) What is at about $+10$ m ?

(b) What is at about -10 m ?

(c) What is about 30 m higher than the chest?

(KS3/95/Ma/Levels 3-5/P1)

3. Write down each number sentence putting in one of the signs, $<$ or $=$ or $>$, to make it correct.
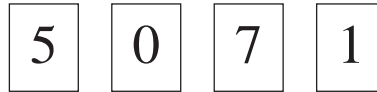
(a) $8 + 2$ ...... $7 + 6$

(b) $6 - 3$ ...... $1 + 2$

(c) 0 ...... -3

(KS3/99/Ma/Levels 3-5/P1)

4. Write the following sums of money in pounds, in decimal form.

(a) Seventy two pounds, forty five pence.

(b) One hundred and three pounds, fifty pence.

(c) One hundred and thirty pounds, five pence.

5.   Here are some number cards:

5	0	7	1

    (a)   What is the *largest* possible number you can make, using all four cards?

    (b)   What is the *smallest* possible number, using all four cards but starting with a non-zero digit?

    (c)   What is the *smallest* possible number you can make, using only three of the cards and starting with a non-zero digit?

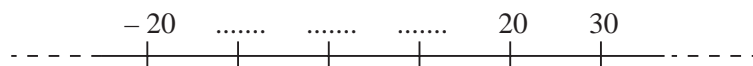6.   (a)   Look at this part of a number line:

          350   400   450   500
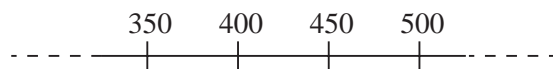
      Copy and complete this sentence:

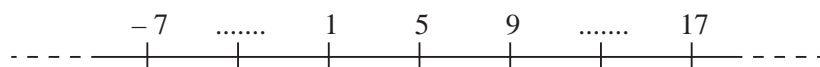   The numbers on this number line go *up* in steps of ....

    (b)   This is a *different* number line.
        What are the 3 missing numbers?

          40   60   ....
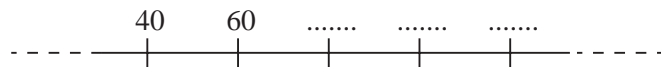
    (c)   This is a *different* number line.
        What are the 3 missing numbers?

          − 20   20   30

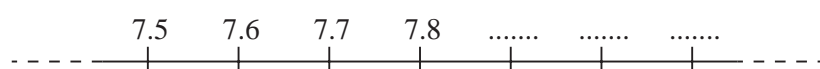   (d)   This is a *different* number line.
        What are the 2 missing numbers?

          − 7   1   5   9   17

      Copy and complete this sentence:

           The numbers on this number line go *up* in steps of ....

    (e)   This is a *different* number line.
        What are the 3 missing numbers?

          7.5   7.6   7.7   7.8   ....

      Copy and complete this sentence:

           The numbers on this number line go *up* in steps of ....

                                      (KS3/97/Ma/Tier 3-5/P1)

2.2 Addition and Subtraction

This section deals with the revision of addition and subtraction of both whole numbers and decimals; we also look again at the use of brackets. You are *not* expected to use a calculator in this section.

Example 1

Calculate:

(a) $1142 + 363$

(b) $4478 - 227$

Solution

(a)
```
  1 1 4 2
+   3 6 3
---------
  1 5 0 5
    1
```

(b)
```
  4 4 7 8
-   2 2 7
---------
  4 2 5 1
```

Note that it is important to *line up* the numbers with the *same place value*.

Example 2

Calculate:

(a) $14 - (8 + 3)$

(b) $16 - (12 - 3)$

Solution

Remember to carry out the calculations in the *brackets first*.

(a) $14 - (8 + 3) = 14 - 11$
$= 3$

(b) $16 - (12 - 3) = 16 - 9$
$= 7$

Example 3

Calculate:

(a) $6.27 + 13.4$

(b) $17.6 - 8.31$

Solution

Remember to *line up* the decimal points.

(a)
```
    6 . 2 7
+ 1 3 . 4 0
-----------
  1 9 . 6 7
```

(b)
```
  0 1   5 1
  1 7 . 6 0
-   8 . 3 1
-----------
    9 . 2 9
```

4. Calculate:

(a) $4.7 - 2.4$ (b) $8.6 - 6.5$ (c) $3.9 - 1.4$

(d) $4.92 - 1.81$ (e) $6.91 - 2.3$ (f) $4.7 - 2.19$

(g) $3.7 - 2.17$ (h) $14.2 - 9.08$ (i) $5.6 - 4.72$

5. Calculate:

(a) $20 - (6 + 2)$ (b) $14 - (8 - 2)$

(c) $18 - (3 + 1)$ (d) $100 - (37 - 22)$

(e) $18 - (11 + 4)$ (f) $22 - (11 + 1)$

(g) $144 - (80 + 12)$ (h) $66 - (5 + 17)$

(i) $100 - (15 - 9)$ (j) $200 - (101 + 42)$

6. Copy the following calculations and fill in the missing numbers:

(a) $962 - \text{.......} = 476$ (b) $\text{......} - 128 = 415$

(c) $3612 = \text{......} + 43$ (d) $7526 = \text{......} - 78$

7. Write one number at the end of each calculation to make it correct:

(a) $400 + 150 = 500 + \text{......}$ (b) $14 + 6 = 4 + \text{......}$

(c) $37 - 20 = 27 - \text{......}$ (d) $38 + 17 = 28 + \text{......}$

(e) $38 - 17 = 28 - \text{......}$ (f) $54 - 26 = 14 + \text{......}$

8. There are 32 pupils in class 7DC, 28 pupils in class 7BD and 29 pupils in class 7PD.
   How many pupils are there altogether in these 3 classes?

9. There are 74 people on a bus. At one stop 22 people get off. How many people are left on the bus?

10. Ben spends £4.27 in one shop and £15.99 in another shop.

(a) How much does he spend altogether?

(b) If he started with £25, how much money does he have left?

11. Bella buys a value burger meal that costs £3.28 for herself and a fun meal that costs £2.25 for her sister.

(a) How much does she spend altogether?

(b) How much change should she get from a £10 note?

12. A triangle has sides of length 18.8 cm, 14 cm and 12.75 cm. Calculate the perimeter of the triangle.

13. Look at these number cards:

$$\boxed{+3}\quad\boxed{0}\quad\boxed{-5}\quad\boxed{+9}$$

$$\boxed{+2}\quad\boxed{-8}\quad\boxed{+7}\quad\boxed{-2}$$

(a) Choose a card to give the answer 4.

$$\boxed{+2} + \boxed{-5} + \boxed{} = 4$$

(b) Choose a card to give the *lowest* possible answer.

Write out the calculation and work out the answer.

$$\boxed{-2} + \boxed{} = \ .....$$

(c) Choose a card to give the *lowest* possible answer.

Write out the calculation and work out the answer.

$$\boxed{-2} - \boxed{} = \ .....$$

(d) Now choose a card to give the *highest* possible answer.

Write out the calculation and work out the answer.

$$\boxed{-2} - \boxed{} = \ .....$$

(KS3/97/Ma/Tier 4-6/P1)

2.3 | Multiplication and Division

In this section we review multiplication and division. Again, you are *not* expected to use a calculator.

Example 1

Calculate:

(a) 41×10 (b) 4.712×100

(c) $62 \div 100$ (d) $23.7 \div 10$

Solution

(a) $41 \times 10 = 410$ (b) $4.712 \times 100 = 471.2$

(c) $62 \div 100 = 0.62$ (d) $23.7 \div 10 = 2.37$

Example 2

Calculate:

(a) 12×24 (b) 37×15

Solution

(a)
```
     1 2
×    2 4
-------
     4 8
   2 4 0
-------
   2 8 8
-------
```

(b)
```
     3 7
×    1 5
-------
   1 8 5
   3 7 0
-------
   5 5 5
-------
     1
```

Note: With all these examples, there are many ways of obtaining the correct answer; for example, in (a) above:

$$
\begin{aligned}
12 \times 24 \ &= \ (10 + 2) \times 24 \\
&= \ (10 \times 24) + (2 \times 24) \\
&= \ 240 + 48 \\
&= \ 288
\end{aligned}
$$

However, we have used the written algorithm for long multiplication as it will *always* work, whereas short-cut methods do not necessarily generalise.

25

Example 3

Calculate:

(a) 4.7×5 (b) 6.4×2.3

Solution

(a)   Since

$$
\begin{array}{r}
4\ 7 \\
\times \quad 5 \\
\hline
2\ 3\ 5 \\
\hline
{\scriptstyle 2\ 3}
\end{array}
$$

   then

$$4.7 \times 5 \;=\; \frac{47 \times 5}{10} \;=\; \frac{235}{10} \;=\; 23.5$$

(b)   Since

$$
\begin{array}{r}
6\ 4 \\
\times \quad 2\ 3 \\
\hline
1\ 9\ 2 \\
1\ 2\ 8\ 0 \\
\hline
1\ 4\ 7\ 2 \\
\hline
{\scriptstyle 1}
\end{array}
$$

   then

$$6.4 \times 2.3 \;=\; \frac{64}{10} \times \frac{23}{10} \;=\; \frac{64 \times 23}{100} \;=\; \frac{1472}{100} \;=\; 14.72$$

Note: When dividing by 10, the decimal point is moved one place to the left; when dividing by 100 the decimal point is moved 2 places to the left, and so on.

Example 4

Calculate:

(a) $124 \div 4$ (b) $615 \div 5$

Solution

(a)
$$
\begin{array}{r}
3\ \ 1 \\
4\,\overline{\smash{)}\,1\ {}^{1}2\ \ 4}
\end{array}
$$

(b)
$$
\begin{array}{r}
1\ \ 2\ \ 3 \\
5\,\overline{\smash{)}\,6\ {}^{1}1\ {}^{1}5}
\end{array}
$$

Again, you can use short-cut methods; for example, in (b) above:

$$615 \div 5 \;=\; 615 \div \left(\frac{10}{2}\right)$$

$$= \left(2 \times 615\right) \div 10 \quad \text{(i.e. dividing by 5 is equivalent to multiplying by 2 and then dividing by 10)}$$

$$= 1230 \div 10$$

$$= 123$$

However, using the *standard method* for division will *always* give the correct answer.

Example 5

A chocolate bar costs 32p. Calculate the cost of 7 chocolate bars.

Solution

```
    3 2
  ×   7
  2 2 4        The cost is  224p  or  £2.24.
  2 1
```

Exercises

1. Calculate:

    (a) 6×10 (b) 17×100 (c) 8×1000

    (d) 14×10 (e) 321×10 (f) 4.2×10

    (g) 3.6×100 (h) 14.7×10 (i) 0.461×100

2. Calculate:

    (a) $4700 \div 10$ (b) $360 \div 10$ (c) $421 \div 10$

    (d) $16.8 \div 10$ (e) $476 \div 100$ (f) $5600 \div 100$

    (g) $56.2 \div 100$ (h) $113.6 \div 100$ (i) $0.652 \div 10$

3. Calculate:

    (a) 15×6 (b) 34×2 (c) 82×7

    (d) 37×5 (e) 19×6 (f) 82×4

    (g) 16×12 (h) 24×14 (i) 32×24

    (j) 66×47 (k) 84×28 (l) 62×29

4. Calculate:

    (a) 4.7×2 (b) 6.3×5 (c) 11.4×5

    (d) 12.7×3 (e) 14.8×4 (f) 22.1×7

    (g) 1.2×3.7 (h) 4.2×5.9 (i) 1.24×1.6

    (j) 7.23×1.4 (k) 18.2×3.2 (l) 27.6×4.2

5. Calculate:

    (a) $12 \div 4$ (b) $81 \div 9$ (c) $42 \div 7$

    (d) $24 \div 8$ (e) $64 \div 8$ (f) $45 \div 5$

    (g) $75 \div 5$ (h) $86 \div 2$ (i) $98 \div 7$

    (j) $128 \div 4$ (k) $248 \div 4$ (l) $497 \div 7$

    (m) $1917 \div 9$ (n) $411 \div 3$ (o) $855 \div 5$

6. Write out each of these calculations, filling in the missing numbers:

    (a) $6 \times \ldots = 120$ (b) $\ldots \div 8 = 7$

    (c) $26 \times \ldots = 962$ (d) $\ldots \div 24 = 16$

7. Write one number at the end of each calculation to make it correct:

    (a) $6 \times 5 = 3 \times \ldots$ (b) $40 \times 10 = 4 \times \ldots$

    (c) $5 \times 30 = 25 \times \ldots$ (d) $7000 \div 100 = 700 \div \ldots$

    (e) $480 \div 20 = 2400 \div \ldots$ (f) $355 \times 12 = 1420 \times \ldots$

8. A packet of crisps costs 32p. Calculate the cost of:

    (a) 3 packets,   (b) 7 packets,   (c) 25 packets.

9. A meal at a burger bar costs £2.95. Calculate the cost of:

    (a) 2 meals,   (b) 3 meals,   (c) 5 meals.

10. Joseph counts the number of sweets in a packet and find that there are 22. How many sweets are there in total in:

    (a) 6 packets,   (b) 100 packets,   (c) 17 packets?

11. Three brothers are given 102 football stickers by their uncle. If they share them equally, how many stickers will they each have?

12. Four children are paid £42.60 for working as gardeners. How much will they each have if they share the money equally?

13. Stamps are 19p each.

    Gwyn wants to buy 9 stamps.

    He knows that he will have to pay *less* than £2.

    (a) Write down how you can tell that he will have to pay less than £2 *without* working out the exact answer.

    (b) Gwyn buys 9 stamps at 19p each.

    Without using a calculator, work out exactly how much he must pay.

14. Gwen makes kites to sell.

    She sells the kites for £4.75 each.

    (a) Gwen sells 26 kites.

    Without using a calculator, work out how much money she gets for the 26 kites.

    (b) Gwen has a box of 250 staples.

    She uses 16 staples to make each kite.

    Without using a calculator, work out how many complete kites she can make using the 250 staples.

15. Here are some buttons on cards.
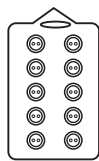
10	5	2
round buttons on a card	star buttons on a card	flower buttons on a card

    (a) Marc bought 9 cards of *star* buttons.

    How many buttons did he buy altogether?

    (b) Lee bought 8 cards of *round* buttons and
                    2 cards of *flower* buttons.

    How many buttons did he buy altogether?

(c)   Sally bought *exactly* 16 buttons.

They were all the *same sort* of button.

What sort of buttons did Sally buy?

(d)   Pat bought *exactly* 15 buttons.

They were all the *same sort* of button.

What sort of buttons did Pat buy?

(e)   Pinder wants to buy *exactly* 20 buttons.

They must all be the *same sort* of button.

Pinder could buy:

        2 cards of *round* buttons.

Write down *two* other possible answers for Pinder.

                                      (KS3/96/Ma/Tier 3-5/P1)

16.   Megan wants to plant 24 seeds.

She can plant them in 2 rows, with 12 seeds in each roll.

● ● ● ● ● ● ● ● ● ● ● ●
● ● ● ● ● ● ● ● ● ● ● ●
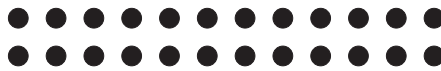
(a)   Draw a diagram to show how she can plant 24 seeds in 3 rows, with the same number of seeds in each row.

(b)   Draw a diagram to show a *different* way that Megan can plant 24 seeds in a *different number* of rows, with the same number of seeds in each row.

(c)   Copy and complete the table to show how many rows Megan can make with 24 seeds, and how many seeds there are in each row.

Number of rows	*Number of seeds in each row*
1 row	24 seeds in a row
2 rows	12 seeds in a row
...... rows	...... seeds in a row
...... rows	...... seeds in a row
...... rows	...... seeds in a row
8 rows	3 seeds in a row
12 rows	2 seeds in a row
24 rows	1 seed in a row

(d)   Megan says:

"I can plant 24 seeds in 5 rows, with the same number of seeds in each row."

Explain why Megan is wrong.

You can write your answer, or draw a diagram.

<div align="right">(KS3/96/Ma/Tier 3-5/P2)</div>

2.4 Problems in Context

Problems in context are dealt with in this section. You will need to decide which operation is required to solve each problem: you may need to *add, subtract, multiply* or *divide*. However, it is still recommended that you tackle these problems without a calculator, perhaps using it only to check your answers.

Example 1

It costs £1.25 for a child to go into a swimming pool. How much does it cost for 7 children to go in?

Solution

(a)
```
        1 . 2 5
    ×         7
    ─────────────
        8 . 7 5          The cost will be  £8.75.
    ─────────────
      1     3
```

Example 2

There are 242 passengers on a train. At a station, 36 people get off and 27 people board the train. How many people are now on the train?

Solution

$242 - 36 + 27 \ = \ 206 + 27$

$\qquad\qquad\quad = \ 233$

So 233 people are now on the train.

Example 3

Four children want to buy a computer game that costs £24.80. How much money
must each of them contribute if they share the cost equally between them?

Solution

```
       6 . 2 0
   4 | 2 4 . 8 0
```
Each child must pay £6.20.

Exercises

1.   A blank tape costs 65p. Calculate the cost of:

     (a)   4 tapes,   (b)   7 tapes,   (c)   9 tapes.

2.   Alec spends £14.27 in a shop. He pays with a £20 note. How much change
     should he get?

3.   The cost of a carpet is £7.99 per square metre. Calculate the cost of:

     (a)   4 square metres,

     (b)   10 square metres,

     (c)   9 square metres.

4.   Simon is saving up to buy a tent that costs £72. So far he has saved £54.50.
     How much more does he need to save?

5.   Two neighbours agree to share equally the cost of a new fence. The fence
     costs £142. How much do they each have to pay?

6.   A cake weighs 824 grams. It is divided into 4 equal parts. How much does
     each part weigh?

7.   A car is driven at a speed of 45 mph. How far does it travel in:

     (a)   2 hours,   (b)   5 hours,   (c)   3.5 hours ?

8.   Cinema tickets cost £7 each. How many tickets could you buy with £63 ?

9.   Cans of drink cost 42p each.

   (a)   How much would 6 cans cost?

   (b)   Jane's mum pays for 6 cans with a £5 note. How much change should she have?

10.   A school trip is arranged for 43 pupils accompanied by 2 teachers. A minibus carries 16 passengers. Three minibuses are booked for the trip. How many empty seats are there in the minibuses?

11.   (a)   A shop sells plants at 95p each.
            Find the cost of 35 plants.

   (b)   The shop also sells trees at £17 each.
            Mr Bailey has £250.
            He wants to buy as many trees as possible.
            How many trees can Mr Bailey buy?

                                                    (KS3/97/Ma/Tier 3-5/P1)

12.   (a)   Lucy had dinner.
            It cost £13.40.
            She paid with a £20 note.
            How much change should Lucy get?

   (b)   (i)   14 people had the set meal at the cafe at a cost of £6.40 each.
                How much did they pay altogether?

        (ii)   Another group of people had the set meal.
                Altogether they paid £32.
                How many people were in the group?

                                                    (KS3/97/Ma/Tier3-5/P2)

13.   Five people shared a bag of apples.

      Each person had the *same number* of apples.

      There were none left

   (a)   How many apples could have been in the bag?

   (b)   Write another number of apples which could have been in the bag.

   (c)   Write another number of apples which could have been in the bag.

The five people shared a box of sweets.

There were more than 100 sweets in the box.

Each person had the same number of sweets.
There were none left.

(d) Anna says: *"I think there were 113 sweets in the box."*

Explain why Anna must be wrong.

(e) Write *two* different numbers of sweets which could have been in the box.

(f) How can anyone tell that your numbers could be divided by 5 just by looking at how they end?
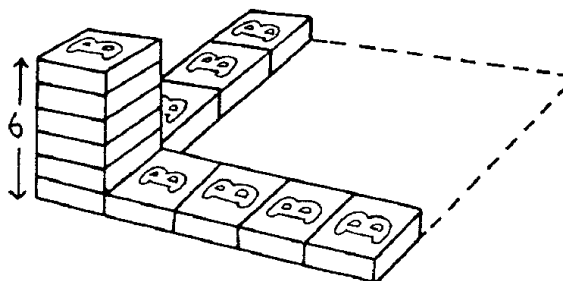
(KS3/94/Ma/Tier3-5/P2)

14. (a) Carl is putting packs of biscuits into a box.
He starts to put in the bottom layer.
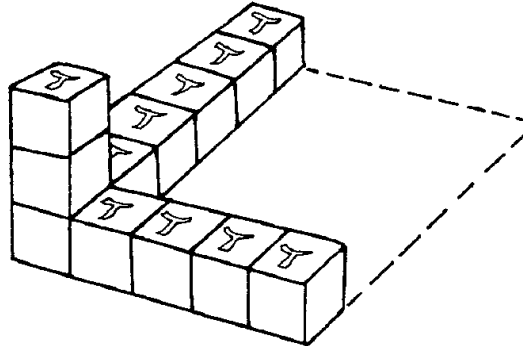The box holds *5 packs across* and is *4 packs wide*.
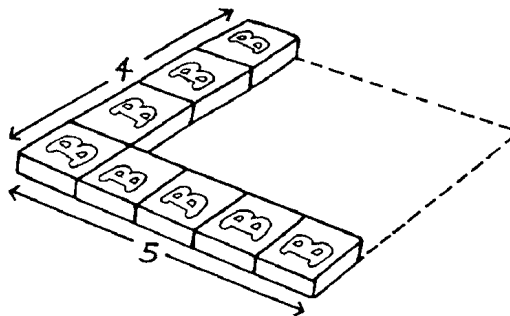
How many packs will fit altogether on the bottom layer?

The box holds *6 layers.*

How many packs will fit in the box when it is *full*?

(b)   Aziz is putting packs of tea into a box.

The box holds *5 packs across* and is *6 packs wide.*

The box holds *3 layers*.

How many packs of tea will fit in the box when it is *full*?

(c)   Copy the words below, filling in the gaps to show one way of filling a *different* box with 24 packs in 2 layers.

total: 24 packs

2 layers

.... packs across

.... packs wide

(KS3/97/Ma/Tier4-6/P2)

15.   (a)   A shop sells video tapes for £2.50 each.
         What is the cost of 16 video tapes?

(b)   The shop sells audio cassettes.
         Each cassette costs £1.49.
         What is the cost of 4 cassettes?

(c)   How many cassettes can you buy with £12?

(d)   The shop also sells cassettes in packs of three.
         A pack costs £3.99.
         How many packs can you buy with £12?

(e)   What is the *greatest number* of cassettes you can buy with £15?
         You can buy some packs and some single cassettes.

(KS3/98/Ma/Tier 3-5/P1)

16. Bill, Ravi and Eric are three divers in a competition.

    Each type of dive has a *dive rating*.

    Easy dives have a *low* rating; *hard dives* have a *high* rating.

    Every dive is marked by five judges who each give a mark out of 10.

    How to calculate the score for a dive:

    1. Look at all five marks. Remove the highest and the lowest marks.
    2. Add together the middle three marks to give a total.
    3. Multiply this total by the dive rating.

    (a) Bill does a dive with a dive rating of 3.34.

    The judges give the marks 7.0   7.5   8 .0   8.0   8.5

    What is Bill's score?

    (b) Ravi scored 82.68 on his first dive.

    The dive had a dive rating of 3.18.

    What was the *total* of the middle three marks given by the judges?

    (c) Eric is getting ready to take his final dive.

    He needs to score at least 102.69 to win the competition.

    Eric decides to do a dive with a dive rating of 3.26.

    Explain why Eric has made a poor decision.

    Show your working.

    (KS3/96Ma/Tier 4-6/P1)

17. A class is planning a trip to a funfair.

    The pupils have found out the prices at these two funfairs:

Milltown Funfair	Seaview Funfair
Entry: £2.20	Entry: £4.50
plus	plus
Rides: 60p each	Rides: 20p each

    The teacher says that there will be time for 8 rides.

    (a) How much money do you need to get in to Milldown Funfair and have 8 rides?

    (b) How much money do you need to get in to Seaview Funfair and have 8 rides?

    Ben has only £5 to get in and pay for his rides.

    (c) How many rides would Ben get at each funfair?

    (KS3/94/Ma/Tier 3-5/P1)